The Solow model at a glance

QM1 Python bootcamp. Companion to the three introductory model slides and the section 7 shocks experiment.

1 · Production and allocation

A closed economy produces one good using capital K_t and workers L_t, with no government or foreign trade. Work **per worker**: $k_t = K_t/L_t$, $y_t = Y_t/L_t$. Baseline productivity is constant, $A_t = A$.

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \quad \Rightarrow \quad y_t = A_t k_t^{\alpha}, \qquad i_t = s y_t, \quad c_t = (1-s) y_t, \quad y_t = c_t + i_t.$$

$A_t > 0$ is productivity; $0 < \alpha < 1$ gives **diminishing returns**: more capital raises output, but each extra unit adds less. The saving rate $0 < s < 1$ is fixed, not chosen by households.

Calibration: $A = 1$, $\alpha = 0.3$, $s = 0.2$, $\delta = 0.1$, $n = 0.02$. Rates are per period. In code, A is `productivity` and α is `alpha`.

2 · Capital accumulation and timing

Depreciation removes a fraction $0 < \delta < 1$ of capital. Today's investment contributes to **tomorrow's capital**. The workforce grows at $n \geq 0$: $L_{t+1} = (1+n)L_t$.

$$K_{t+1} = \underbrace{(1-\delta)K_t}_{\text{retained capital}} + \underbrace{sY_t}_{\text{investment}}, \qquad \boxed{k_{t+1} = \frac{(1-\delta)k_t + sA_t k_t^{\alpha}}{1+n}}.$$

Divide tomorrow's capital by tomorrow's workforce: population growth dilutes capital per worker. Given $k_0 > 0$ and a productivity path, repeat the update; recover output and consumption at each date.

3 · The deterministic steady state · sections 2–4a

With **constant productivity**, capital rises or falls according to

$$k_{t+1} - k_t = \frac{sAk_t^{\alpha} - (\delta + n)k_t}{1+n}.$$

At a steady state, $k_{t+1} = k_t = k^*$: investment covers depreciation and population dilution, $sA(k^*)^{\alpha} = (\delta + n)k^*$. The positive solution is

$$k^* = \left(\frac{sA}{\delta + n}\right)^{1/(1-\alpha)}, \qquad y^* = A(k^*)^{\alpha}, \quad c^* = (1-s)y^*.$$

Below k^*, capital rises; above it, capital falls. Positive starts approach the same k^* for the same parameters; starting at zero stays at zero. Baseline: $k^* \approx 2.07456$, $y^* \approx 1.24474$, $c^* \approx 0.99579$.

Comparisons: higher s or A raises k^*, y^*; higher δ or n lowers them. Higher saving reduces consumption immediately at unchanged capital and need not raise steady consumption.

Growth: with constant A, growth per worker is transitional; aggregate output can still grow with population.

4 · Repeated productivity shocks · section 7

Replace constant productivity with independent positive draws:

$$\boxed{A_t = \bar{A}\exp(\sigma\varepsilon_t - \sigma^2/2)}, \qquad \varepsilon_t \overset{\text{iid}}{\sim} N(0,1), \qquad E[A_t] = \bar{A}.$$

$\bar{A}$ is mean productivity (`productivity`); $\sigma \geq 0$ controls shock size. The correction fixes the **theoretical mean**, not each sample mean. There is no technological trend or change in the saving rule.

- **Timing:** inherited k_t is unchanged by A_t; output changes now and capital at $t+1$. Final A_T affects y_T, c_T without another stored capital update.
- **Experiment:** 150 updates / 151 observations; start at `k_analytical`; compare $\sigma = 0, 0.1, 0.3$ using the same normal draws. At $\sigma = 0$, $A_t = \bar{A} = 1$.
- **Interpretation:** the deterministic steady state is a reference, not an asserted stochastic mean or convergence target under continuing shocks. Use the chosen horizon.

Source: Sargent and Stachurski, "The Solow–Swan Growth Model", QuantEcon, sections 27.1–27.2. This course retains population growth n; their baseline holds the workforce constant.